Abstract Algebra HW #1

Dr. Day

Due September 6th, 2019

- Reading: Read all of Chapter I (sections 1.1–5.5).
- Practice: Attempt but do not turn in the following: 2.10, 2.11, 3.1, 4.1, 4.4, and 5.2.
- Turn in: 2.5, 3.1, E1 (below), 4.3, and 5.9.

Problem E1: Recall that a *partial order* on a set A is a binary relation (usually denoted $\leq$) on A that is reflexive, antisymmetric, and transitive (look up these terms if they are unfamiliar). A *partially ordered set* (or *poset*) is a set together with a partial order on it. A function $f\colon A \to B$ between two partially ordered sets is *order preserving* if, whenever $x, y \in A$ with $x \leq y$ in A, then $f(x) \leq f(y)$ in B.

(a) Define a category C as follows: $\mathrm{Obj}(\mathsf{C})$ is the class of all partially ordered sets, and for any $A, B \in \mathrm{Obj}(\mathsf{C})$, $\mathrm{Hom}_{\mathsf{C}}(A, B)$ is the set of all order preserving functions from A to B. Define composition of morphisms in C to be composition of functions. Prove that C is a category.

(b) Now fix a partially ordered set $(A, \leq)$. Define a category A with $\mathrm{Obj}(\mathsf{A}) = A$ and for any $a, b \in A$, $\mathrm{Hom}_{\mathsf{A}}(a, b)$ being $\{(a, b)\}$ if $a \leq b$ and $\varnothing$ if $a \not\leq b$. Since nonempty homsets are singletons, there is a unique way to define composition for A. Prove that A is a category.

(c) Is there a special connection between these two constructions?