Fall 2019 Math 3083 Test #1 Practice Test

Dr. Day's section 002

The format of the practice test is similar to the format of the test, but in the real test there will space for your work and your answers.

Rules: The test has a 50 minute time limit. The test is closed-book and closed-notes (no cheat sheet). Only non-programmable calculators may be used (and these are optional); no cell phones, computers, or other devices may be used.

Advice: Please don't leave blanks. Writing a definition or a description of how to do something can be worth partial credit. Please pace yourself: allow about a minute for every two points on a problem. Skip ahead if you get stuck and come back to hard problems later. Please show your work. Please read each question carefully and answer what it asks!

1. True/false with optional justification. For each item, select a true/false answer. You may justify your answer by giving an example or writing an explanation, if you like. If your true/false answer is wrong, you may get partial credit based on your justification.

   (a) (4 points) Suppose n is a positive integer. If A and B are invertible $n \times n$ matrices, then so is AB.
   True False
   Justification:

   (b) (4 points) Suppose n is a positive integer and A is an $n \times n$ matrix. If A is row-equivalent to the identity matrix, then the determinant of A must be 0.
   True False
   Justification:

   (c) (4 points) It is possible for an underdetermined linear system of equations to have exactly one solution.
   True False
   Justification:

   (d) (4 points) Suppose $A_1, \ldots, A_n, B_1, \ldots, B_n$ are matrices such that both sides of the following equation make sense. Then the equation holds.

   $$\left(\begin{array}{c|c|c|c} A_1 & A_2 & \ldots & A_n \end{array} \right) \begin{pmatrix} B_1 \\ \hline B_2 \\ \hline \vdots \\ \hline B_n \end{pmatrix} = A_1B_1 + A_2B_2 + \ldots + A_nB_n$$

   True False
   Justification:

2. (16 points) Put the following matrix in reduced row-echelon form:

$$\begin{pmatrix} 1 & 1 & 3 & -2 \\ 1 & 2 & 4 & -2 \\ -1 & 1 & -1 & 2 \end{pmatrix}$$

3. (10 points) The following linear system is already in reduced row-echelon form. Solve it (parametrize all possible solutions, for example by writing out the solution set).

$$\begin{cases} x_1 - x_2 + 3x_3 + x_5 = 0 \\ x_4 + \frac{1}{3}x_5 = 1 \\ x_6 = 0 \end{cases}$$

4. For each of the following matrices, find the inverse, or determine that the matrix is not invertible. If the matrix is invertible, compute the product of the matrix with its inverse to check your work.

(a) (10 points)

$$\begin{pmatrix} 4 & 3 \\ 6 & 5 \end{pmatrix}$$

(b) (24 points)

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

5. (24 points) Evaluate the following determinant, using whatever methods you like.

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 6 & 8 \\ 3 & 6 & 10 & 12 \\ 4 & 8 & 12 & 18 \end{vmatrix}$$

Answers:

1.(a) True. (b) False. (c) False. (d) True.
2. Use row reduction, showing your work. The answer is:

$$\begin{pmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

3. $\{(\alpha - 3\beta - \gamma, \alpha, \beta, 1 - \frac{1}{3}\gamma, \gamma, 0) \in \mathbb{R}^6 | \alpha, \beta, \gamma \in \mathbb{R}\}$. (There are other ways of writing this.)
4.(a) Find the inverse using the formula, then check your answer. The row reduction method is also acceptable. The answer is:

$$\frac{1}{2}\begin{pmatrix} 5 & -3 \\ -6 & 4 \end{pmatrix}$$

(b) Find the inverse using row reduction, then check your answer. The formula using the adjoint matrix is also an acceptable method, but is not recommended. The answer is:

$$\begin{pmatrix} -3 & 1 & -2 \\ -1 & 0 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

5. The answer is 2. Row reduction is the recommended method. Cofactor expansion is acceptable, but is a lot more work.